The Fundamental Behaviours of the Electron are Reproduced by a Mechanical Vortex Model

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Abstract

Vortex configurations have long been used to describe the structure of elementary particles. This paper demonstrates that the physical characteristics of the electron emerge naturally from a leapfrogging pair of vortex rings, an arrangement which demonstrates a formal correspondence with topological soliton solutions found in gauge field theories. Exact derivations are found for the Bohr magneton and related magnetic formulae. Coulomb's law is recovered as a formal isomorphism with the Bjerknes acoustic radiation force and electric permittivity is thereby identified as the compressibility of the vacuum medium. From this single measured constant the complete hydrogen energy spectrum follows as a necessary consequence: the Bohr radius, the ground state binding energy of 13.6 eV, and the Lyman and Balmer spectral series are all reproduced exactly. The angular structure of atomic orbitals is independently recovered through the multipole correspondence between acoustic radiation modes and orbital angular momentum quantum numbers. The Larmor radiation catastrophe is resolved directly: topologically protected flows in a superfluid vacuum retain quantised energetic states in equilibrium with their medium. The electron's fundamental properties are thus derived from purely geometric constraints. The mechanical Onsager-Feynman equation predicts the electron's Compton radius which in turn predicts the value of magneton. These results are consistent with quantum mechanics, QFT, and the Standard Model, while providing a mechanical foundation for phenomena that currently lack causal explanation.

Keywords: Vortex Dynamics, Topological Solitons, Superfluid Vacuum, Fine Structure Constant, Secondary Bjerknes Forces

Method

Knotted and linked vortex lines have been created in classical fluids, superfluids, Bose-Einstein condensates, plasmas and optical fields, where they can persist as robust structures rather than fleeting disturbances. Newberry's first paper "A Demonstration That an Electron Consists of a Ring Vortex Pair, with Greater than 7 Sigma Statistical Support..." [1] demonstrated characteristic matching between the electron and leapfrogging ring vortices, referred to here as co-linked vortices or CLVs, establishing their equivalence in probabilistic terms. This paper adds further detail to the model and as well as numerical verification.

Hopf type field solitons are vortex structures which support sustained internal dynamics, including oscillatory energy exchange and topologically linked configurations analogous to CLVs [2,3]. Notably, the same Hopf description can describe vortices in superfluids, since the description does not distinguish between different realisations of the underlying field, a characteristic first made explicit in the Skyrme-Faddeev model [4]. Most recently Eto et al. demonstrated that knotted solitons emerge as stable constructs and indicate that they provide a model for the neutrino [5].

The CLV is stabilised through kinematic phenomena: Arnold tongue locking, a mechanical property of tori, fixes the circulation ratio between toroidal and poloidal flow, which combined with the circulating sheath determines both the scale and longevity of the configuration [7]. The equivalence arises because vortex solutions in the Gross-Pitaevskii equation are topological defects of the underlying U(1) gauge symmetry, making the stability results from superfluid systems directly applicable to the gauge-theoretic setting [4,7]. The CLV therefore sits within current standard gauge theory and admits a physical realisation in a superfluid vacuum substrate.

A common and correct critique of vortex ring particle models is that they lack any mechanism to fix their scale and require continuous translational motion to persist. However, as Wang et al. showed in 2015, the addition of toroidal rotation produces a centrifugal force which, when phase-locked with the poloidal rotation, allows the structure to persist without the movement through a medium [6]. The Biot–Savart description of CLVs is applicable to Bose-Einstein condensates (BECs) but it does not fully capture the situation within energetic superfluids; hydrodynamic modelling indicates that the rotational velocity fields around the rings act as effective velocity capture mechanisms [7]. Within this energetic medium, only high-momentum fluctuations possess sufficient inertia to penetrate the centrifugal barrier to the vortex core, whereas lower-momentum components are diverted by the sheath flow. This filtration mechanism is known as inertial segregation in vortex dynamics [8]. Consequently, smaller CLV systems with higher rotational frequencies ($\omega \propto 1/r$) exhibit a more efficient coupling with the high-energy components of the vacuum, allowing them to maintain a higher total energy density than larger, lower-frequency configurations. This is confirmed by Finne et al who showed that two distinct regimes exist: one dominated by damping and the other persisting via energy entrainment [9]. In the CLV context, the high frequency of smaller structures ensures they remain deep within this inertial regime, creating a steeper pressure gradient that facilitates the energy cascade.

Experimental and theoretical studies of vortices and solitons with winding number greater than one show that they are unstable and split into singly quantised structures under small perturbations [8]. In three-dimensional settings where the elementary defects are closed vortex rings, this instability leads to the formation of two coaxial, co-propagating singly charged rings. Numerical studies of vortex ring dynamics in the Gross-Pitaevskii equation show that such ring pairs naturally enter a leapfrogging regime, providing an understanding of how CLVs form without fine-tuning [9,10].

In the CLV framework, half integer spin emerges from the coupled dynamics of the leapfrogging vortex rings and their surrounding fluid sheath. This behaviour is rooted in a geometric relationship known as the Coin Paradox where two half rotations of two coins around each other return the coins to their original positions, but with their design inverted. It is also a behaviour characteristic of topological solitons in a vacuum substrate. Consequently, a full 720° rotation is required to return the system to its initial physical state [10]. An additional cause would arise if there exists a 2 to 1 ratio between the toroidal and poloidal rotations.

The mathematical presentations of the Hopfion have to be based on some physical precursor as opposed to their being purely numerical entities. As Feynmann said, “Mathematicians are only dealing with the structure of reasoning, and they do not really care what they are talking about... But the physicist has meaning to all his phrases. That is a very important thing that a lot of people who come to physics by way of mathematics do not appreciate. Physics is not mathematics” [11]: we must reach beyond mathematics to arrive at physics and vortex structures require a fluidic substrate to exist, a vortex is by definition an organised rotation of something, and this requirement applies equally whether the medium is classical, superfluid or described by a gauge field. Dirac, Einstein and many others have shared this conviction that the vacuum possesses a physical structure. As many readers will be aware, the choice of a superfluid is a natural one due to the deep formal isomorphisms between electromagnetic field equations and fluid dynamics, a relationship noted as far back as Maxwell [12].

Table 1 Some general formulaic correspondences between electromagnetic and fluid dynamic environments.

Property	Hydrodynamic (CLV) Representation	Electromagnetic (Maxwell) Representation
Field Variable	Velocity Field v	Vector Potential A
Topological Core	Vorticity $\nabla \times v = \omega$	Magnetic Field $\nabla \times A = B$
Conserved Quantity	Kinetic Helicity $H = \int v \cdot \omega \, d^3x$	Magnetic Helicity $H_m = \int A \cdot B \, d^3x$
Dynamic Energy	Kinetic Energy Density $\frac{1}{2}\rho v^2$	Magnetic Energy Density $(1/2\mu_0)B^2$
Interaction Force	Acoustic Force $(\rho / 4\pi) (\dot{V}_1\dot{V}_2 / d^2)$	Coulomb Force $F_c = (1/4\pi\varepsilon_0)(q_1q_2/d^2)$

The Wave Function

The mapping between the CLV and the Madelung formulation of the Schrödinger equation follows directly from the hydrodynamic interpretation of the wavefunction. The description

$$\psi = \rho^{05} e^{i(s/\hbar)}$$

separates the Schrödinger equation into a continuity equation for ρ and a Hamilton-Jacobi equation for S, where ρ represents a physical fluid density and $\nabla S/m$ defines a velocity potential [13].

In conventional treatments, the associated quantum potential Q is often regarded as an abstract or non-classical term. Within the CLV framework Q admits a direct mechanical interpretation as an internal pressure gradient associated with the vortex structure. Just as a classical vortex is stabilised by a low pressure core that generates restorative stresses, the quantum potential represents the hydrodynamic resistance of the leapfrogging rings to compression or expansion of their configuration. Explicitly:

$$Q = -\,(\hbar^2 / 2m)\,(\nabla^2\rho^{05} / \rho^{05})$$

Rather than a formal correction, this term corresponds to the restorative energy required to maintain the CLV's specific topological state against external perturbations. Sharp spatial variations in the density field generate stabilising stresses, encoded by the Laplacian relative to the local density, the tighter the vortex configuration, the stronger the restoring force.

This interpretation extends directly to the observable density ρ, which in the CLV model corresponds to the mass energy distribution of the surrounding sheath. In free space, the sheath forms the effective external interface of the electron and adopts a smooth, approximately Gaussian profile as the medium redistributes high frequency core motion outward [13]. Spatial derivatives of ρ therefore encode the curvature and non uniformity of this interface, providing a mechanical basis for the curvature dependent terms.

In this picture, the wavefunction constitutes a coarse grained description of a fluid interface rather than a fundamental probability wave. Hydrodynamic and stochastic analogues of quantum mechanics independently confirm that wave like behaviour emerges from deterministic dynamics mediated by an underlying medium [15].

Coupled oscillatory systems admit stable anti phase locking, producing stationary patterns through fixed node to antinode attraction. Electrons accordingly aggregate into patterns and locations dictated by this phase locking mechanism, a prediction empirically confirmed by Scanning Moiré Fringes, the node riding observed in the Borrmann Effect, and the spatial corralling of electrons into whispering gallery modes within confined geometries [14–16]. Stationary interference patterns of this kind are characteristic signatures of wave propagation through a physical medium and their existence is difficult to account for by other methods.

The CLV model resolves the ambiguity between wave phase and mechanical oscillation by grounding Zitterbewegung directly in the dynamics of the vortex core. In the standard Dirac formulation, Zitterbewegung is the interference between positive and negative energy states, oscillating at $2mc^2/h$ [17]. The CLV framework identifies this frequency with the rhythmic, high speed energy exchange between the leapfrogging rings, providing a mechanical realisation of the spinor phase. As the rings cycle, the vortex topology returns periodically to its initial state. The electron clock is therefore the fundamental cycle of the vortex geometry rather than a stochastic jitter. Hestenes reaches the same conclusion through Space Time Algebra: the Dirac phase factor represents a physical circulation, a periodic state of the vacuum substrate, rather than an abstract degree of freedom for a point particle [18].

Taking the Zitterbewegung frequency and applying the coin paradox spin relation, the classical electron radius is recovered from the thin ring radius a of a vortex leapfrogging at c. This derivation yields a/R equal to the fine structure constant, a precise geometric result that is not inserted by hand but emerges necessarily from the kinematics. That the Zitterbewegung frequency itself awaits direct experimental detection is noted.

Stability and Radius

The CLV is maintained through the balance between Lighthill radiation which represents the acoustic loss to the medium and the energy gain of the rotational sheath via velocity selection. The extended surrounding flow, very marked in superfluids, further stabilises this process by acting as a dynamical buffer, damping short-term fluctuations and mediating interactions [21]. Through this exchange, the structure emerges as a self-regulating structure whose scale, and subsequent particle identity, is dictated by the energy density of the vacuum. The balance of inflow and outflow obviates the historical criticism of mechanical models which entail that the electron loses energy through Larmor radiation [19]. This universality has a precise topological counterpart: in the Skyrme-Faddeev model, the energy of a Hopf soliton with topological charge $Q_H = 1$ possesses a unique energy minimising configuration [4,24,25], so both the fluid mechanics and the field theory independently enforce a unique, stable configuration.

The thin filament radius, derived above, shapes the behaviour of electrons as they are observed in collisions. The model predicts that at high velocities, the rings will not deflect from each other, rather, that they will either destroy each other or cut through each other and mend themselves in a phenomenon known as vortex reconnection [2]. Furthermore, with the protective sheath and with the gyroscopic inertia, collision experiments at high energies are predicted to fail at measuring the electron radius [26].

The frequently cited 10^{-22} m upper limit on the electron radius (Dehmelt 1988) is not a direct measurement of size [21]. It is an extrapolation from the measured g-factor using the Brodsky-Drell relation, which assumes a composite model in which point-like sub-constituents orbit a common centre [22]. The deviation of g from 2 would scale as $(r/\lambda_C)^2$ in such a model. Since the CLV electron is not composed of independent pointlike sub-particles orbiting each other, but is a topological defect in the medium whose magnetic moment arises from the total circulation of the structure, the Brodsky-Drell relation does not apply.

Charge

The CLV model replaces the abstract concept of charge with a tangible mechanical process: the radial acoustic pulsations generated by the leapfrogging rings and, where present, the Kelvin waves. Acoustic radiation forces are the only known mechanism in any field of physics capable of generating attractive forces at a distance. As the rings rotate at the Compton frequency and leapfrog, they act as approximate spherical resonators, recovering the Coulomb inverse square law directly [23,24].

The leapfrogging motion imposes a characteristic plus plus pressure waveform on both rings simultaneously. Two CLVs of identical topology therefore always lock into an anti-phase state with respect to each other and make repulsion, between like CLVs, a structural inevitability rather than an assumption. Radiative damping suppresses asynchronous modes that would otherwise lead to structural instability, driving interacting CLVs toward stable equilibrium [28]. This explains why like charge carriers exhibit mutual repulsion while remaining decoupled from neutral matter [26].

A difficult conundrum, from a physics perspective, is the construction of two particles of different sizes, frequencies and energies, e and p, which nevertheless produce exactly the same force at a distance on one another. A naive reading suggest that a reduction of the core radius ‘a’ reduces the volumetric displacement by a squared factor, whilst the pulsation frequency only increases linearly as ‘a’ decreases, a mismatch which would make a force independent of size impossible as the terms do not cancel. However, the Bjerknes force depends on the second time derivative of the volumetric displacement, $\ddot{V}$, which scales with f^2 rather than f. The squared frequency term exactly compensates the squared reduction in displaced volume, yielding an acoustic pressure field that is independent of core radius. This is the mechanical origin of charge universality: a result that in standard physics must be assumed and goes unexplained, whilst here it emerges as a necessary consequence of the acoustic force law.

The high bulk modulus of the vacuum medium further ensures tight acoustic coupling between CLVs, analogous to sympathetic resonance between tuning forks in air, but in a medium where propagation losses are negligible and impedance mismatch is absent. Since induced in-phase acoustic coupling produces attraction, this mechanism provides the physical basis for proton to electron attraction: the observed Coulomb force between unlike charges is the macroscopic signature of this resonant entrainment.

The vacuum permittivity ε_0 is thereby identified as the electromagnetic counterpart of acoustic compliance: in any acoustic medium, the force scale of the radiation pressure interaction is set by ρc^2, the bulk modulus of the medium. In air or water, varying ρc^2 directly scales the acoustic force between pulsating sources, precisely as varying ε_0 scales the electrostatic force between charges. The structural correspondence is exact; ε_0 encodes the same physical property of the vacuum medium that ρc^2 encodes in classical fluids.

The dimensional mismatch between ρc^2 and $1/\varepsilon_0$ reflects the historical introduction of the coulomb as an independent unit — a necessary consequence of treating electric charge as a primitive quantity rather than a derived mechanical one. Within the CLV framework, charge is identified with $\dot{V}$, a purely mechanical quantity with dimensions m^3/s^2, and the coulomb is accordingly a derived rather than fundamental unit.

Magnetic Moment

The mathematical identity between the Biot Savart law for vortex filaments and for magnetic fields dates to Helmholtz (1858) and was central to Maxwell's original formulation of electromagnetism, in which he explicitly equated magnetic field strength with vorticity and magnetic permeability with the density of his vortex sea [27]. In the CLV framework, this identity is not an analogy but a physical correspondence: the magnetic field of a charged particle is the velocity field of its vortex structure. The CLV model yields the correct derivation of the magnetic moment: a charge e circulating at R at c produces the Bohr magneton value [29]. In physical terms this is the leapfrogging motion at c producing a pumping action to produce the moving field. The force is seen in action in superfluids where vortices form coherent paired and bundled structures that persist over large distances. In addition vortex-antivortex pairing has been directly observed in turbulent BECs [28].

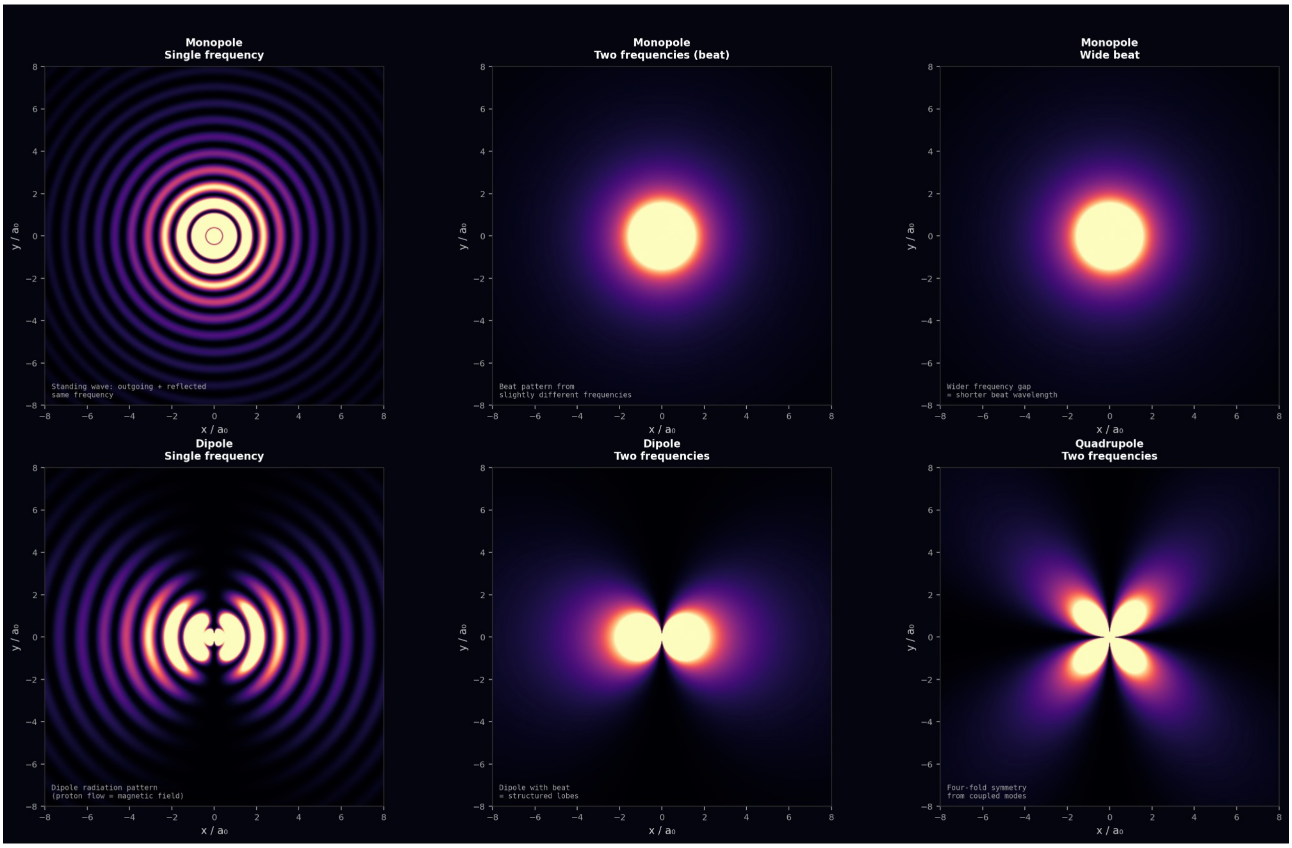
The Onsager-Feynman circulation quantisation formula applied to the CLV as a single co-linked topological structure with circulation $\Gamma = h/m_e$ and surface velocity c, yields $R = \hbar/m_e \cdot c$ exactly. This is the reduced Compton wavelength. The same radius R, with charge e circulating at c via the leapfrogging motion, gives $\mu = e\hbar/2m_e$ the Bohr magneton exactly. These two results emerge simultaneously from one geometric constraint. This constitutes a significant independent validation of the CLV model: a formula from established superfluid vortex physics, applied without modification, derives the electron's scale and magnetic moment as simultaneous necessary consequences.

A further consistency check is provided by applying the Biot-Savart translational velocity formula to the CLV. In a superfluid the classical core radius used in the Biot-Savart formula is replaced by the healing length $\xi = \hbar/m\_e \cdot c \cdot \sqrt{2}$, the physically correct cutoff for a quantised vortex in a superfluid [29,30]. With this substitution the logarithmic term reduces to $\ln(8\sqrt{2}) \approx 2.4$, and the translational velocity of the leapfrogging rings comes out approximately equal to c. The corresponding leapfrogging frequency $f = U/2\pi R$ then approximates the electron Compton frequency $m_e \cdot c^2/h$, giving, from $E = hf \approx m_e \cdot c^2$, the electron rest mass energy to within 9%. The Biot-Savart formula carries inherent approximation and, if present, Kelvin waves also affect the translational velocity and if included would act in the direction to reduce this residual discrepancy further [31,32]. The result demonstrates that the CLV model is consistent with the de Broglie-Einstein relation $E = hf$ again connecting the physics of vortices with the results from particle physics, and provides a further approximate verification for the framework.

Hydrogen Orbitals

Janusson et al. (2020) established a mathematical isomorphism between hydrogen orbital shapes and acoustic standing wave patterns. By demonstrating that an identical differential equation governs both systems, they proved that quantum numbers are not merely analogous to, but are functionally identical to acoustic mode numbers [32]. In the CLV framework, the proton establishes an acoustic field in the surrounding superfluid whose resonance modes reproduce the nodal structure of the hydrogen orbitals. The electron CLV is localised within the acoustic potential wells generated by this field, occupying force equilibrium positions that correspond directly to the observed orbital geometries.

Fig 1 Proton Acoustic Emission: Time-Averaged Intensity (p^2)



Showing nodes produced by acoustic formulae. Time-averaged acoustic intensity $\langle p^2 \rangle$ from a CLV source configuration representing the proton, computed from the acoustic wave equation in spherical coordinates. The displayed quantity is:

$\langle p^2\rangle = (1/T)\int_0^T p^2(r,\theta,t)\,dt$ where the pressure field $p(r,\theta,t)$ is the superposition of outgoing and reflected spherical waves with angular modulation determined by the emission geometry: $p = (A/r)\cdot F(\theta)\cdot \cos(kr - \omega t)$. Here $F(\theta)$ is the multipole angular factor (monopole, dipole or quadrupole depending on emission mode), $k = \omega/c$ is the wavenumber derived from the CLV Compton frequency, and r is distance from the source. Bright regions indicate high acoustic intensity. The spatial scale is set by the proton CLV geometry with $R \approx 0.72$ fm. Medium properties ρ and c are pinned by the vacuum permittivity equivalence established in the text.

Table 2. Exact reproductions of known formulas and constants obtained through the natural mechanics of the CLV when it is given the Compton frequency and the Classical Electron Radius.

Physical Quantity	CLV Geometric Input	Resulting Expression	Status / Accuracy
Ring Radius R	Onsager-Feynman circulation quantisation using surface velocity c	$R = \hbar/m_e \cdot c$	Exact derivation — reduced Compton wavelength
Core Radius (a)	Classical electron radius	$a = e^2 / (4\pi\varepsilon_0 mc^2)$	Derived from Dirac f
Mag. Moment (μ)	Charge circulating at speed c	$\mu = e\hbar / (2m)$	Exact Value
Permittivity ($1/\varepsilon_0$)	Impedance Speed Identity	$1/\varepsilon_0 = Z_0\, c$	Exact Identity
Coulomb Force	Acoustic Radiation Force	$F = (\rho/4\pi d^2)\, \dot{V}_1\dot{V}_2^*$	Accurate Isomorphic Match
Phase Symmetry	Leapfrog topology	Half spin	Exact Value
Hydrogen Energy Spectrum	Acoustic resonance modes of proton field	$E_n = 13.6\ \mathrm{eV}/n^2$, Bohr radius, Lyman and Balmer series	Exact
Leapfrogging Frequency	Biot-Savart translational velocity with superfluid healing length	$f_{leap} \approx f_{Compton} =$ m_e·c²/h	Approximate — within 9% of Compton frequency; Kelvin wave corrections act to reduce residual discrepancy

*$\dot{V}$ denotes the volumetric flow rate of the source (the time derivative of the displaced volume). For a two-ring CLV, $\dot{V}$ serves as the acoustic analogue of electric charge, with multiple co-located sources adding linearly.

This paper identifies a substantial number of mathematical correspondences and physical indicators that collectively establish the vacuum as a physical acoustic medium:

The correspondence between hydrogen orbital shapes and acoustic standing wave patterns

The Madelung formulation of quantum mechanics is a fluid dynamical description of the wavefunction.

Scanning Moiré Fringes, the Borrmann Effect, and whispering gallery modes are direct signatures of wave propagation through a physical medium.

The Biot Savart law is mathematically identical for vortex filaments and magnetic fields, as recognised by Maxwell, who explicitly identified magnetic permeability with the density of a vortex medium.

The speed of light is identified as the speed of sound of the vacuum medium.

Electron diffraction requires a medium through which waveforms propagate.

The only known mechanism by which attractive forces arise at a distance without direct contact is acoustic radiation pressure.

As Volovik states: 'the physics of the vacuum is the physics of a medium' [33].

These correspondences are substantial but not an exhaustive list [1,12]. Standard objections to a superfluid vacuum typically address uniform media they do not engage with the dynamics of topologically protected vortices, which exhibit unique stability and invariance properties that specifically address the historical objections to mechanical models.

The appearance of ħ as the circulation quantum in the Onsager-Feynman condition suggests a deeper identification. In a superfluid, the vacuum wavefunction must be single valued — a topological constraint that forces circulation into integer multiples of h/m, with no stable sub-quantum rotation possible. If the vacuum is such a medium, ħ is not an independently measured constant but the minimum circulation quantum of the medium itself. The discreteness of quantum mechanics — the universal appearance of ħ as a minimum unit of angular momentum — is then a direct consequence of vacuum topology rather than a postulate. The fine structure constant α appears in the CLV geometry as the ratio a/R — the ratio of the microstructural scale of the vacuum to its continuum quantum length scale — suggesting it is a material property of the medium rather than a free parameter. Future work may productively investigate whether α can be derived from first principles as the acoustic coupling ratio between the vacuum medium and its topological defects, in which case c, ħ and α would together constitute a complete mechanical description of the vacuum: c setting the velocity scale, ħ setting the rotational granularity, and α setting the coupling strength between the medium and the structures it supports.

Conclusion

This paper complements Eto et al.'s recent demonstration that knotted solitons emerge as stable solutions representing neutrinos [5]. Both papers establish that stable topological particle structures arise naturally within gauge theoretic frameworks. This paper has further shown that the electron's mechanical characteristics are derived from the proposed structure. As some of these aspects are complex, the hydrogen orbitals and the production of forces at a distance among them, the structural parallels constitute independent verification of the model's validity. Beyond the spin value, the Bohr magneton, and the Coulomb force constant, full numerical matches between the geometric parameters and the measured electron properties are not yet achievable, as very little data beyond energy and charge is currently known about the electron's internal structure.

Treating the electron as a hydrodynamic entity opens a direct path for computational fluid dynamics simulations of quantum behaviours, including the double slit experiment, providing an independent numerical test of the correspondence between classical vortex dynamics and quantum observation.

This mechanical picture also provides the foundation for the mass energy equation

$$E = mc^2 = V\rho v k$$

where V is the volume of the vortex system, v is the internal average speed, ρ is the medium density, and k is the propagation speed of disturbances in the medium [1]. Dimensional consistency requires k to carry units of velocity; in a relativistic vacuum this corresponds to the invariant signal speed c.

The CLV model produces a combination of features that is rarely achieved: stable mass, convergence with the Schrödinger equation, and a coherent mechanical framework for the relevant forces. No other single framework reproduces this combination. The model accounts for electron behaviours that are both complex and distinctive, making their unification within a single classically based framework particularly compelling.

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